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Experimental studies of stability and transition in high-speed wakes

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The investigation undertaken deals with the development of disturbances in a supersonic wake (free viscous layer and regular wake) behind a flat plate both in its linear and nonlinear stages. The influence of a number of factors (Mach and Reynolds numbers, temperature factor, thickness of the plate, length of its stern) on the wake stability and transition was studied. The development of the artificial disturbances in a wake at Mach number $M_\infty = 2$ was investigated also.

It was found that compressibility of the flow (increasing Mach number) stabilizes the wake disturbances – their amplification rates decrease, and the transition point moves away from the model plate. Cooling of the model surface at $M_\infty \sim 7$ has a destabilizing influence on the development of disturbances in the wake. With increase of unit Reynolds number the beginning of transition in the wake moves forward to a rear critical point. It was confirmed that a distinctive maximum in the spectral distribution of fluctuations appears, corresponding to Strouhal number (based on frequency of this maximum) of 0.3. With the growth of the model thickness the disturbance amplification rates in the wake increase, which results in earlier transition of a laminar wake into turbulent one. With the growth of length of the plate stern, the position of the wake transition moves back accordingly, while the wake stability increases a little (though very unsignificantly). In the nonlinear stage of development of disturbances, the occurrence of a triad of waves, satisfying the resonant correlation of frequencies, and the growth of harmonics are observed. A monochromatic packet of waves of Tollmien–Schlichting type, rather narrow (in the transversal coordinate) in the boundary layers on a flat plate with an opposite wedge at the stern, was found to extend in the wake. The wake disturbances have a complex wave structure. At the Mach number of free flow 2.0, the three-dimensional disturbances are the most unstable in the wake.

1. Introduction

Research on the flow in the wake behind an object is an important problem of aerodynamics. The base drag on rotation-shaped bodies at supersonic speeds can account for up to 30% of their complete drag (and in particular for cones up to 50%; Mihalev 1980; Kovenya & Lebedev 1989), i.e. it largely determines the aerodynamics of the flying body. In addition, the value of base drag can increase by more than twice from laminar to turbulent regimes (Mihalev 1980).

The problem of transition from laminar to turbulent flow in wakes and jets is of great practical significance for decreasing noise generation in the jets of rocket and aircraft engines, at supersonic conditions, and for improvement of the mixture of fuel and oxidizer flows inside the chamber of combustion of a hypersonic flying body.

However there have not been many experiments on the stability of a wake at supersonic flow speeds carried out so far (similar studies in a boundary layer are more numerous – see, for example, Lysenko 1993; Lysenko & Maslov 1984; and many others), though such works as Behrens (1968), Behrens & Ko (1971), Behrens, Lewis & Webb (1971), Demetriades (1964, 1978, 1990), McLaughlin (1971), McLaughlin *et al.* (1971) have become classical. In them the development of the natural disturbances is studied.

While there are many works in which the development of artificial disturbances in a supersonic boundary layer and a jet have been studied (see § 6 of the present paper), there is only one similar study (Martens, Kinzie & McLaughlin 1994) in a shear layer (mixing layer). Work on the development of artificial disturbances in a supersonic wake is not yet available.

The present paper is devoted to an experimental study of the stability and transition in the supersonic wake behind a flat plate. Section 2 contains results on the transition position in supersonic and hypersonic wakes at the free-flow Mach numbers $M_\infty = 4$, 5 and 7, different unit Reynolds numbers and different values of the temperature factor. Section 3 is devoted to the study of the stability of a wake at $M_\infty = 4$, § 4 to the influence of some parameters of supersonic free flow on the development of disturbances in a wake (at $M_\infty = 2$ and 4 and different unit Reynolds numbers), § 5 to the influence of the thickness of the flat plate and the length of its stern on the stability of a wake at $M_\infty = 2$. And the last part (§ 6) contains a study of the development of artificial disturbances (initiated on the surface of the flat plate) in the system comprising the boundary layer on the model and the wake, at $M_\infty = 2$.

This work is the result of a series of investigations, taking several years.

2. The transition position in supersonic and hypersonic wakes: influence of temperature and other factors

The experiments were carried out in the hotshot tunnel 'Transit' at Mach numbers $M_\infty = 4$, 5 and 7 for a large range of unit Reynolds number $Re_{1\infty} = (40\text{–}150) \times 10^6\,\mathrm{m}^{-1}$. A symmetric flat plate of 108 mm length and 10 mm thickness, the nose being a wedge with a half-angle of bevel 14°, was used as a test model. The plate stern was blunt (bevelled to a right angle). The plate was fixed rigidly to the lateral walls of the wind tunnel test section and was exposed at zero angle of attack. By changing the temperature (heating) of the air in the stilling chamber the experiments were performed at different values of the temperature factor (wall-to-adiabatic wall temperature ratio) $T_w = 1$, 0.85, 0.8 and 0.6.

To fix the transition position in the wake and in the boundary layer of the plate a Pitot tube was used with a total-head-tube reception aperture of 0.3 mm height and 1.3 mm width (external sizes of the tube were 0.4 mm and 1.4 mm) and diameter of the reception aperture of the static-pressure probe of 0.3 mm, and induction probes DMI, dialog-computer complex DVK-2M and the high-speed digital system of registration 'Spectrum-2'. To define the position of the boundary-layer transition, the location of the total-head tube on the model surface was changed along the longitudinal coordinate x, and the position of transition was defined in the classical way (i.e. considering the change in the probe signal, depending on the change of speed profiles at transition of a laminar boundary layer to turbulent one; the position of the end of the transitional zone was accepted as the position of transition). To determine the location of

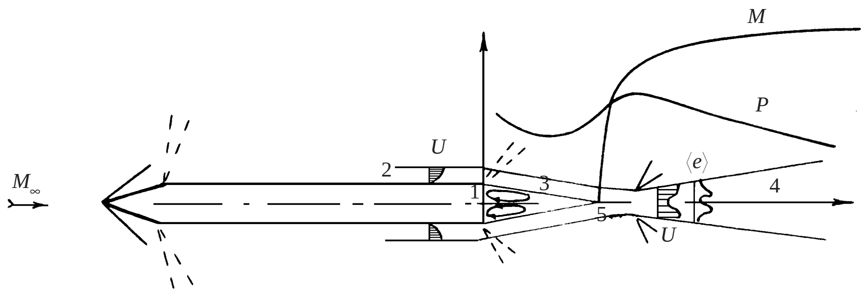

FIGURE 1. Diagram of the model: 1, plate end; 2, boundary layer; 3, free viscous layer; 4, wake; 5, rear critical point.

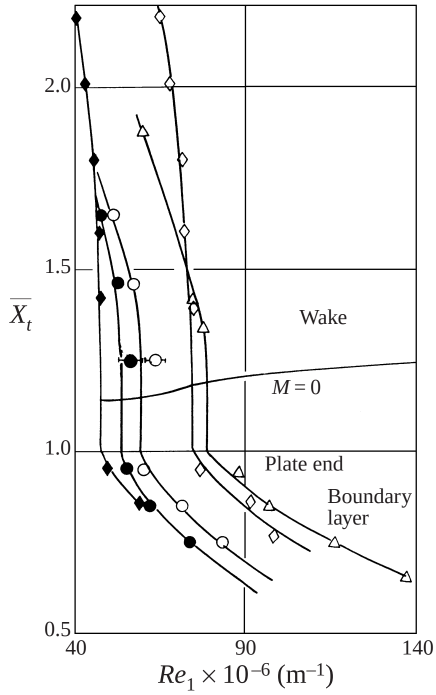

FIGURE 2. Non-dimensional longitudinal coordinate of transition as a function of unit Reynolds number (●, ○, $M_\infty = 4$; △, $M_\infty = 5$; ◇, ◆, $M_\infty = 7$; ●, $T_w = 1$; ○, △, $T_w = 0.8$; ◇, $T_w = 0.85$; ◆, $T_w = 0.6$.

transition in the wake, the position of the total-head tube along the wake longitudinal axis was varied, and the beginning of significant growth of the probe signal, connected with the change of speed profiles at transition of a laminar wake to turbulent one (decreasing Mach-number defect), was accepted as the beginning of transition (also according to the classical way – see, for example, McLaughlin *et al.* 1971).

In figure 1 a schematic of the flow around the model in the experiments is shown. And the results are presented in figure 2. Here $\bar{x}_t = x_t$ (mm)/108 is the non-dimensional longitudinal coordinate of transition. The curve $M = 0$ corresponds to the position of the rear critical point of the wake (the curve was obtained by approximating the experimental data at $M_\infty = 4$ and 5).

The results obtained show that the condition (laminar, transitional or turbulent) of the boundary layer at the end of the plate affects (through the change of profiles of speed and other parameters, and also at the expense of changes of the level of disturbances) the position of wake transition, though the processes of transition (in particular, the amplification of disturbances) can occur simultaneously both in the boundary layer, and in the wake, because of their respective instabilities. The author got the impression that, when decreasing Re_1, while the boundary layer at the model end remained turbulent, the start of wake transition could be near the rear critical point (somewhere in the region of the wake throat), and as soon as Re_1 met the requirements of the transitional condition of the boundary layer, the start of transition in the wake began moving away from the rear critical point, corresponding to decreasing Re_1. Incidentally, while in the case of the transitional boundary-layer condition the stabilizing effect of the boundary layer on the wake is through the change of profiles of speed and other parameters, in the case of the laminar boundary-layer condition the stabilization occurs by means of reducing the disturbance amplitudes.

It is also shown that compressibility (the increase of Mach number) has a stabilizing influence on the wake. For $Re_{1\infty} \approx 60 \times 10^6\ \mathrm{m}^{-1}$ on increasing M_∞ from 4 up to 5 and from 5 up to 7 the position of the beginning of transition moved away from the model end correspondingly. A similar movement of transition in the wake was observed in the wind tunnel T-325 (these data will be shown in §4, figure 9). The results obtained correspond to those of Mihalev (1980), Wen (1964), Lykoudis (1966), Legner & Finson (1977), Roshko & Fishdon (1969), Demetriades (1978), in which the Reynolds number of transition in a wake was found to increase with the increase of Mach number.

Also, it was found that decreasing the temperature factor at $M_\infty = 4$ and $T_w > 0.8$ stabilizes the boundary layer and, as a consequence, the near wake (at least for the range of Re_1 at which the condition of the boundary layer at the end of the model changes), though in the case of a laminar boundary layer on the whole plate and large values of $\bar{x}_t$ the situation can become different. At $M_\infty = 7$ and $T_w < 0.85$ (when in a boundary layer the transition is defined by the second unstable mode) the decrease of temperature factor destabilizes both the boundary layer and the wake. The results obtained do not contradict the theoretical works on stability, in which a decreasing temperature factor destabilized disturbances of both the first mode in a shear layer (Jackson & Grosch 1988, 1989, 1990*a,b*, 1991; Ragab & Wu 1989; Hegde & Zinn 1991; Kudryavtsev & Solovyov 1989, 1991) and a wake (Lees 1964; Lees & Gold 1964), and the second mode in a shear layer (Ragab & Wu 1989*a,b*). However, note that in the work by Ragab & Wu (1989*b*) the influence of cooling of stagnated flow was ambiguous (at $M_1 > 1.5$ it resulted in stabilization of the first mode and destabilization of the second one).

The moving of the position of transition in the wake away from the model end on decreasing the unit Reynolds number was confirmed. A similar influence of Re_1 was observed in the wind tunnel T-325 (these data will be presented in §4, figure 11). This change of the dependence of $\bar{x}_t$ upon Re_1, obtained in the wake (reduction of $\bar{x}_t$ with the growth of Re_1) corresponds to the results by McLaughlin (1971) and McLaughlin *et al.* (1971) ($M_\infty = 4.3$), in which it was found that with the increase of unit Reynolds number the beginning of transition in the wake moves forward to the rear critical point. A similar result was obtained by Pallone, Erdos & Eckerman (1964) for hypersonic ($6710\,\mathrm{m\,s^{-1}}$) flow speed.

It is shown also that the growth of unit Reynolds number leads to the increase of the longitudinal size of the zone of return flow (in Kovenya & Lebedev 1989 and

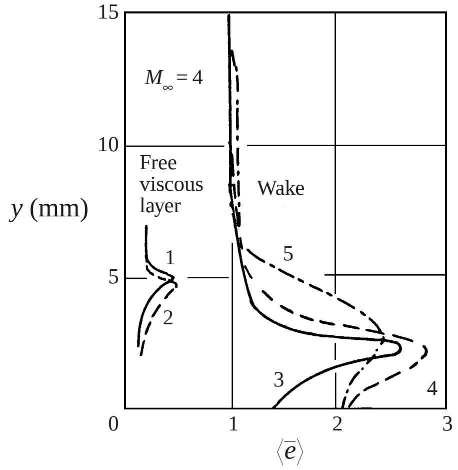

FIGURE 3. Non-dimensional profiles of root-mean-square hot-wire fluctuations in the free viscous layer (curve 1, $x = 12.5$ m; 2, 15 mm) and in the wake (3, $x = 40$ mm; 4, 60 mm; 5, 80 mm) at $M_\infty = 4$.

Reeves & Lees 1965 a similar dependence was obtained for a laminar wake). In the experiments described here the initial reduction of static pressure was precisely fixed at when the rear critical point moved to the back of the model (with subsequent increase in front of the model), defining the air flowing into the recirculating zone.

3. Stability of the wake at Mach number $M_\infty = 4$

These experiments were carried out together with N. V. Semionov in the wind tunnel T-325 (Bagaev *et al.* 1972) at the free-flow Mach number $M_\infty = 4$ and unit Reynolds number $Re_{1\infty} = 9 \times 10^6\ \mathrm{m}^{-1}$. The flow stagnation temperature was about 290 K.

For measurement of the characteristics of stability and transition a constant-current hot-wire anemometer TPT-4 with the probe of tungsten wire of 6 microns in diameter and 1.2 mm long, a selective amplifier U2-8, a voltmeter V7-27/1 and a spectrum analyzer from 'Bruel and Kjaer' (type 2010) were used. An insulated steel symmetric flat plate of 88 mm length, 10 mm thickness and 200 mm width, the nose being a wedge with bevel half-angle of 14° and leading-edge bluntness of 0.1 mm, was used as the test model. The plate stern was blunt (bevelled to a right angle). The plate was fixed rigidly to the lateral walls of the test section of the wind tunnel and was placed at zero angle of attack.

Figure 3 shows the non-dimensional profiles of root-mean-square fluctuations of voltage in the hot-wire-anemometer probe $\langle \bar{e} \rangle(y)$ in the free viscous layer (free jet boundary layer) at values of the longitudinal coordinate of $x = 12.5$ and 15 mm and in the wake (regular wake) at $x = 40$, 60 and 80 mm (the longitudinal coordinate x was measured from the stern of the plate; and for clarity of comparison $\langle e \rangle(y)$ at $x = 12.5$ and 15 mm were non-dimensionalized by $\langle e \rangle_\infty$ at $x = 40$ mm).

It was found that for each x the normal coordinate y at which the value of $\langle e \rangle$ is maximum across the wake corresponds approximately to a turning point in the curve of average voltage in the hot-wire-anemometer probe $E(y)$. And after the analysis of the data, with use of calculated profiles of temperature, the connection of maximum

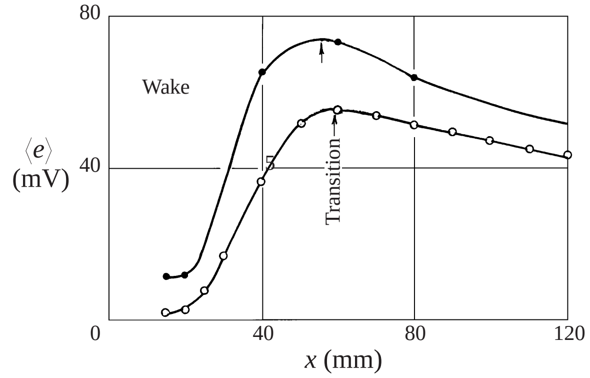

FIGURE 4. Distribution of the hot-wire fluctuations along the longitudinal coordinate at $M_\infty = 4$ and $Re_{1\infty} = 9 \times 10^6\,\mathrm{m}^{-1}$ (arrows show the wake transition position: ●, $y = y_{\langle e \rangle max}$; ○, $y = 0$).

(across a wake) disturbances in the critical layer with a generalized turning point (where $\mathrm{d}/\mathrm{d}y[(1/T)\mathrm{d}u/\mathrm{d}y] = 0$) was confirmed.

The position of transition in a wake is defined rather precisely by the position of the maximum of voltage fluctuations in $\langle e \rangle(x)$ (as, for example, found by Demetriades 1990), and it is similar to the determination of the position of boundary-layer transition using a hot-wire anemometer. In figure 4 such dependences, determined in the experiments here, both in the plane of the wake symmetry ($y = 0$) and in the layer with maximum $\langle e \rangle$ across the wake (this layer is close to the critical one), are shown. The position of transition, determined in this way, in the present case corresponds to $x_t = 56$–$59\,\mathrm{mm}$ (and $\hat{x}_t \approx 40-44\,\mathrm{mm}$ correspondingly for the longitudinal coordinate $\hat{x}_t$ measured from the beginning of the regular wake; $Re_t = (u_e - u_o)\hat{x}_t/\nu_o \approx 0.22 \times 10^5$, where the indexes o and e correspond to the plane of symmetry of the wake and its edge). Moreover in the layer close to the critical one, transition occurs earlier (at $x_t \approx 56\,\mathrm{mm}$) than in the plane of symmetry of the wake (where $x_t \approx 59\,\mathrm{mm}$) (the same situation is described in Behrens 1968).

After wake transition (i.e. at $x > 56\,\mathrm{mm}$) the profile of disturbances in the wake (figure 3) along the longitudinal coordinate become more flat faster than before transition, at the expense of decreasing the maximum disturbances in the wake. The thickness of the most disturbed part of the wake and the normal coordinate of the layer with the maximum disturbances increase more intensively (than before transition), which corresponds to the statements by Behrens (1968) and Demetriades (1964) that appreciable expansion of a wake is an attribute of transition.

The free viscous layer (free jet boundary layer) changes from being similar to a boundary layer just behind the plate to a more-or-less developed mixing layer (which corresponds to the results of Papageorgiou 1991). Therefore it was of particular interest to compare the characteristics of the stability of a free viscous layer with the similar characteristics of a boundary layer, and subsequently with the characteristics of stability of a wake. In figure 5 the spectra of energy of fluctuations in the hot-wire-anemometer probe (distribution of disturbance amplitude $\bar{e}_f$ for different frequencies f) in the free viscous layer for two locations $x = 12.5$ and $15\,\mathrm{mm}$ are shown (here and in § 3 the amplitudes of disturbances for different frequencies are non-dimensionalized by the amplitude of disturbances with frequency 3 kHz at $x = 12.5\,\mathrm{mm}$). In figure 6 the variation of the spatial disturbance amplification rates $-\hat{\alpha}_i = (\mathrm{d}e_f/e_f)\delta/\mathrm{d}x$, in the free viscous layer, with the frequency of disturbance f for $x = 13.8\,\mathrm{mm}$ (curve

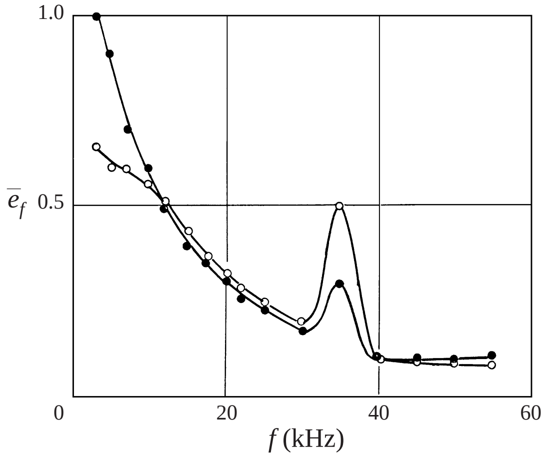

FIGURE 5. The energy spectra of hot-wire fluctuations in the free viscous layer at $M_\infty = 4$ and $y = y_{\langle e \rangle max}$ (●, $x = 12.5$ mm; ○, 15 mm).

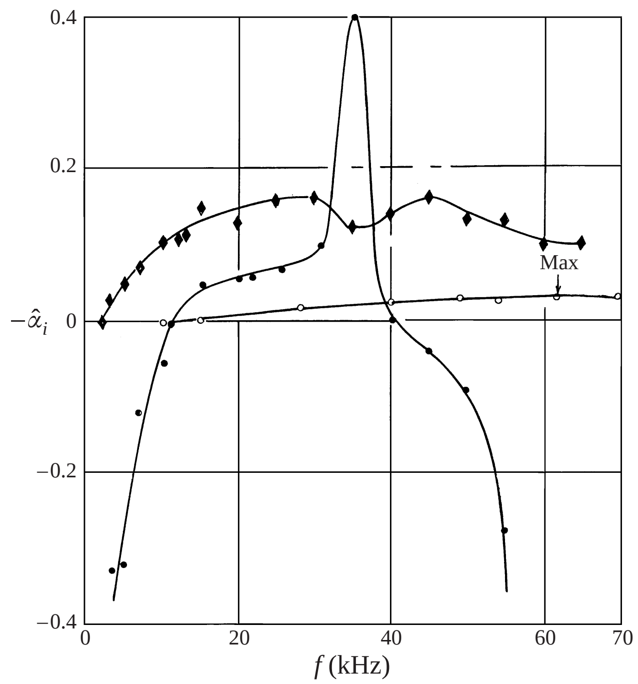

FIGURE 6. The disturbance amplification rates as a function of frequency in the free viscous layer (●, $x = 13.8$ mm), the plate-end boundary layer (○) and the wake (◆, $x = 42.5$ mm) at $M_\infty = 4$, $Re_{1\infty} = 9 \times 10^6\,\mathrm{m}^{-1}$ and $y = y_{\langle e \rangle max}$.

1), and also, for comparison, disturbance amplification rates in the boundary layer at the end of the plate (obtained before for a similar flow over a flat plate, curve 2), are given; here δ is the thickness of the free viscous layer and correspondingly the boundary layer. This dependence of the amplification rates for the boundary layer has the meaning of a second zero (on the top branch of the neutral stability curve) at $f = 140$–150 kHz. As seen from figure 6, though the maximum amplification rates

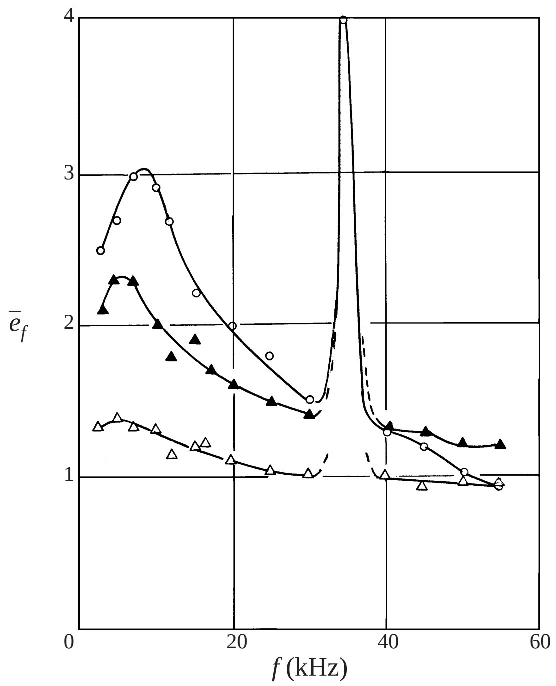

FIGURE 7. The energy spectra in the wake at $M_\infty = 4$ and $y = y_{\langle e \rangle max}$ (○, $x = 40$ mm; ▲, 60 mm; △, 80 mm).

of disturbances in the free viscous layer are higher than in the boundary layer at the end of the plate, the range of 'unstable' frequencies decreases (at the expense of high frequencies), sometimes significantly. And the disturbances which are maximum in the boundary layer at the end of the plate and ready to lead to transition in the boundary layer (recall that, in the boundary layer on a flat plate at $M_\infty = 4$ and $Re_{1\infty} = 9 \times 10^6\,\mathrm{m}^{-1}$ the beginning of transition, measured by the total-head tube, occurs at $x \approx 80$–90 mm, which just corresponds to the end of the model plate tested here), are stabilized in the free viscous layer (these disturbances have $f \geqslant 60$ kHz). The low-frequency disturbances, the most amplified in the free viscous layer, penetrate into it from the plate boundary layer, and, being extremely small, do not have time (up to the throat of the wake) to develop to the 'transitional' level. And in this sense, from the point of view of transition development in the boundary layer–free viscous layer–wake system, the free viscous layer is stable. The data obtained correspond to the statements of Reeves & Lees (1965) that in a viscous free layer (up to the throat of a wake) at $M > 3$ there is laminar flow.

The development of disturbances in the wake (regular wake) is shown in figures 7 and 8, where the spectra of energy of fluctuations, measured in the layer close to the critical one (with the maximum level of disturbances), and in the plane of symmetry of the wake $y = 0$, are presented (at $x = 40$, 60, 80 mm and 40, 50, 60 mm). In figure 6 the amplification rates $-\hat{\alpha}_i$ of disturbances in the wake depending on the frequency of disturbances at $x = 42.5$ mm were included also. And they were compared with the amplification rates of disturbances in the free viscous layer (at $x = 13.8$ mm) and in the boundary layer at the end of the plate considered earlier. While at transition from the boundary layer to the free viscous layer the range of 'unstable' frequencies

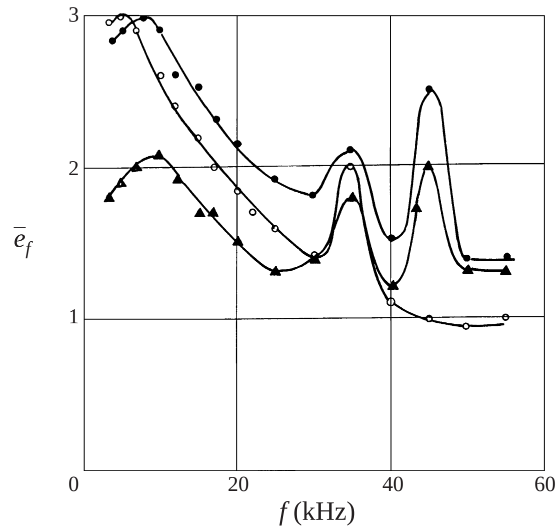

FIGURE 8. The energy spectra in the wake at $M_\infty = 4$ and $y = 0$ (○, $x = 40$ mm; ●, 50 mm; ▲, 60 mm).

decreases significantly (at the expense of high frequencies), at transition from the free viscous layer to the wake this range increases (also at the expense of high frequencies).

Both in the free viscous layer (figure 5), and in the wake in the layer close to the critical one (figure 7), a distinctive maximum in the spectral distribution of fluctuations was found – disturbances with $f \approx 35$ kHz begin to grow significantly. The Strouhal number, based on the frequency of this maximum, the thickness of the wake near the throat (determined from the speed profile) and the speed of the undisturbed flow, is $S = fb_o/u_\infty = 0.3$. The same occurrence of a maximum in the spectrum of fluctuations was observed by Behrens & Ko (1971) for the wake of a flat plate at $M_\infty = 6$. Moreover, Behrens & Ko have also obtained $S = 0.3$, and have shown that this value of the Strouhal number is universal both at subsonic and at hypersonic flow speeds.

The data in figures 4, 7 and 8 suggest that at $Re_{1\infty} = 9 \times 10^6\ \mathrm{m}^{-1}$ the disturbances in the wake (beginning from the throat of the wake) develop linearly up to $x \approx 40$ mm, then there is nonlinear development, and at $x \approx 56$–59 mm transition occurs. The power spectra vary accordingly. And while for the layer close to the critical one (figure 7), the basic tone with $f_0 \approx 35$ kHz dominates in the spectrum of energy in the linear and nonlinear zones (similar to Zhuang, Kubota & Dimotakis 1990), in the layer $y = 0$ (figure 8) the situation is more complex. In the linear stage the disturbances with $f_0 \approx 35$ kHz grow more intensively than the other ones (but to a considerably smaller extent than in the critical layer); however in the nonlinear stage the disturbances with $f_2 \approx 45$ kHz begin to grow quickly. Most likely, the two maxima, at $f_1 \approx 5$–10 kHz and $f_0 \approx 35$ kHz, in the spectrum of fluctuations at $x \approx 40$–45 mm in the critical layer and in the layer $y = 0$, are in resonance, and a triplet of waves arises (specifically, the third wave has $f_2 \approx 45$ kHz). As seen from figure 8, in the layer $y = 0$ the growth of disturbances with frequencies corresponding to the two (at $x = 40$ mm) maxima in the spectrum is much slowed down on moving from $x = 40$ mm to $x = 50$ mm, while the disturbances with $f_2 \approx 45$ kHz grow very much. Thus, in the triplet of waves with frequencies f_0, f_1 and f_2 the transfer of energy goes from the first and second waves (at least, from the basic tone with f_0)

to the third one. It is interesting that, in Behrens & Ko (1971), in the layer close to the critical one there was one maximum only (corresponding to the maximum with $f_0 \approx 35$ kHz in the present work), and in the layer $y = 0$ in Behrens & Ko (1971) only a harmonic with frequency $2f_0$ grew significantly (this difference from the present data is explained, most likely, by the fact that in Behrens & Ko (1971) the thickness of the plate was only 0.38 mm, and it had a sharp back edge, i.e. the recirculating zone was practically absent). Probably, in the present experiments in the layer $y = 0$ the disturbances with frequency $2f_0 \approx 70$ kHz grew but this was not seen, since the measurements were limited to the range of frequencies 3–60 kHz.

Also from figures 7 and 8 it is seen clearly that in the linear stage of the wake disturbances with low frequencies (as a whole, not including a distinctive maximum) prevail, in the nonlinear stage disturbances with higher frequencies grow more intensively, but at transition disturbances with lower frequencies decrease more significantly than high-frequency ones, and the spectrum become more flat. The redistribution of energy results in the energy of fluctuations being levelled for all frequencies.

Thus, at Mach number $M_\infty = 4$ the development of disturbances in the free viscous layer and the wake (regular wake) behind a flat plate with a symmetric wedge-shaped nose with sharp leading edge and blunt stern bevelled to a right angle has been investigated. The characteristics of the stability of the flow in the free viscous layer and the wake have been obtained. The occurrence of a distinctive maximum in the spectral distribution of fluctuations, corresponding to a Strouhal number (based on frequency of this maximum) of 0.3, was found. In the plane of symmetry of the wake in the nonlinear stage of development of disturbances a triad of waves, satisfying the resonant correlation of frequencies, was observed.

4. Influence of the parameters of supersonic free flow on the development of disturbances in a wake

The experiments described in this Section were performed in the wind tunnel T-325 at free-flow Mach numbers $M_\infty = 2$ and 4 and unit Reynolds numbers $Re_{1\infty} = (5.7;$ 9 and $15) \times 10^6\,\mathrm{m}^{-1}$. The flow stagnation temperature was about 290 K.

For measuring the characteristics of stability and transition the constant-temperature hot-wire anemometer K-109 (in several experiments at $M_\infty = 4$ the hot-wire anemometer TPT-4) with a tungsten probe with wire of 6 microns in diameter and 1.2 mm length, a selective amplifier U2-8, a voltmeter V7-27A/1 and a spectrum analyzer from Bruel and Kjaer (type 2010) with level recorder (type 2307) were used.

An insulated steel symmetric flat plate of 61 mm length, 10 mm thickness and 200 mm width, the nose being a wedge with the half-angle of 14° and leading-edge bluntness of 0.1 mm was used as a test model (this plate had practically the same geometrical size as in § 3). The stern of the plate was blunt, of rectangular form. The model was fixed rigidly to the lateral walls of the test section of the wind tunnel and was placed at zero angle of attack.

In figure 9 the dependence of the root-mean-square fluctuations of the voltage of the hot-wire-anemometer probe on the longitudinal coordinate in the plane of symmetry of the wake for $M_\infty = 2$ and 4 at $Re_{1\infty} = 9 \times 10^6\,\mathrm{m}^{-1}$ are shown (the voltage oscillations were normalized by their maxima). One can see that on decreasing the Mach number from 4 to 2 the position of transition approaches the model significantly.

In figure 10 the dependence of the disturbance amplification rates in the wake $-\hat{\alpha}_i = (\mathrm{d}e_f/e_f)b/2\mathrm{d}x$ (b is the wake thickness) on the frequency f for $M_\infty = 2$ and

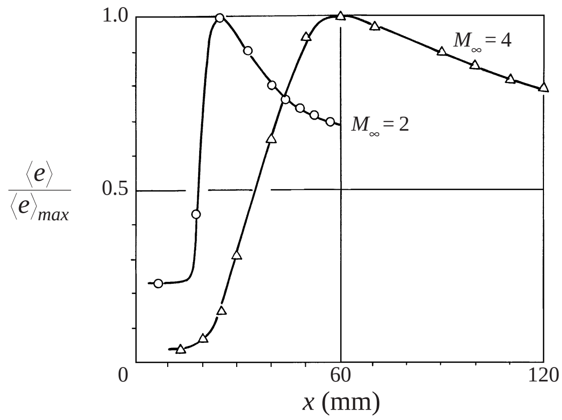

FIGURE 9. Distribution of the hot-wire fluctuations along the longitudinal coordinate at $y = 0$ and $Re_{1\infty} = 9 \times 10^6\,\mathrm{m}^{-1}$ for $M_\infty = 2$ and 4.

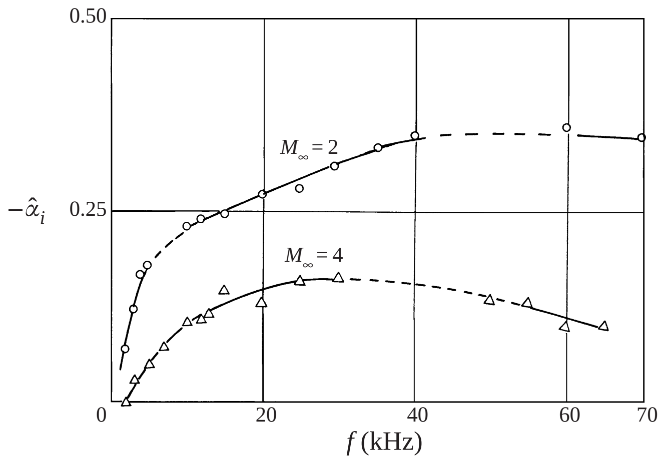

FIGURE 10. The disturbance amplification rates as a function of frequency at $y = y_{\langle e\rangle_{max}}$ for $M_\infty = 2$ and 4.

4 is shown. The measurements were carried out in the layer with maximum (across the wake) values of $\langle e\rangle$ (this layer is close to the critical one). In the figure it is seen that on decreasing the Mach number from 4 to 2 the disturbance amplification rates increased to a great extent.

The growth of the longitudinal coordinate of transition x_t on increasing M_∞ from 2 up to 4 described here corresponds to the growth of x_t corresponding to the increase of M_∞ from 4 up to 5 and from 5 up to 7 obtained in § 2 in the hotshot tunnel 'Transit' in the wake behind a model similar to that used here (in 'Transit' on changing M_∞ from 4 up to 7 for $Re_{1\infty} \approx 60 \times 10^6\,\mathrm{m}^{-1}$ the longitudinal transition coordinate approximately doubled).

Thus, the experiments at $M_\infty = 2$, 4, 5 and 7 have shown the stabilizing influence of the growth of the Mach number on the development of disturbances in a wake. With increasing Mach number (from 2 up to 4) the disturbance amplification rates decrease significantly. This conclusion does not contradict the results of a number of theoretical studies on the stability of mixing layers (Jackson & Grosch 1988, 1989,

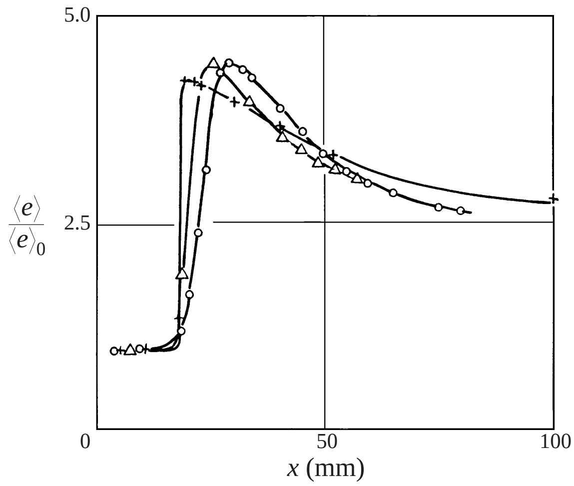

FIGURE 11. Distribution of the hot-wire fluctuations along the streamwise coordinate at $M_\infty = 2$ and $y = 0$ for different unit Reynolds numbers (○, $5.7 \times 10^6\,\mathrm{m}^{-1}$; △, $9 \times 10^6\,\mathrm{m}^{-1}$; +, $15 \times 10^6\,\mathrm{m}^{-1}$).

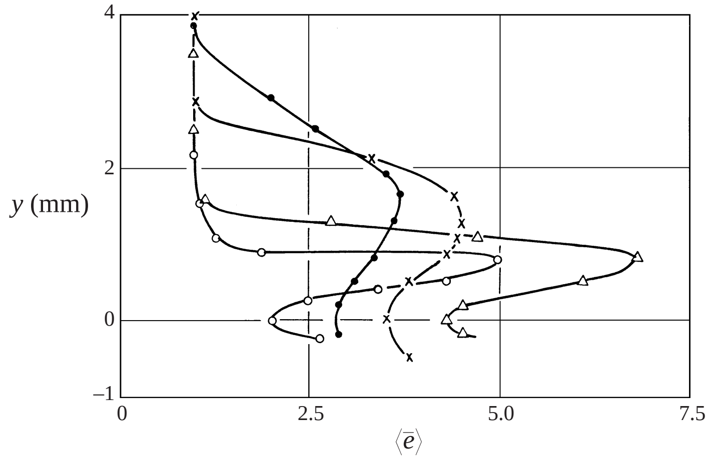

FIGURE 12. Non-dimensional profiles of fluctuations for $M_\infty = 2$ and $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$ at different longitudinal coordinates (○, $x = 20\,\mathrm{mm}$; △, 27 mm; ×, 44 mm; ●, 60 mm).

1990*a,b*, 1991; Kudryavtsev & Solovyov 1989, 1991; Ragab & Wu 1989*b*; Zhuang *et al.* 1990; Kim 1990; Macaraeg & Streett 1989), in which it was shown that at fixed T_w an increase of M increases the stability of the flow in a shear layer in respect to subsonic disturbances. This conclusion corresponds to the theoretical work on the stability of wakes (Lees 1964; Lees & Gold 1964; Gertsenstein & Koshko 1972; Chen, Cantwell & Mansour 1990), in which it was found that with the growth of Mach number the disturbance amplification rates decrease.

The influence of unit Reynolds number on the development of disturbances in the wake was also investigated. The experiments were carried out in the plane of symmetry of the wake at $M_\infty = 2$ and $Re_{1\infty} = (5.7; 9 \text{ and } 15) \times 10^6\,\mathrm{m}^{-1}$, curves 1–3 respectively in figure 11; all voltage fluctuations are non-dimensionalized by their initial values near the edge of the recirculating zone. The stabilization of disturbances (the reduction of amplification rates) and the withdrawal of the transition position in the wake with

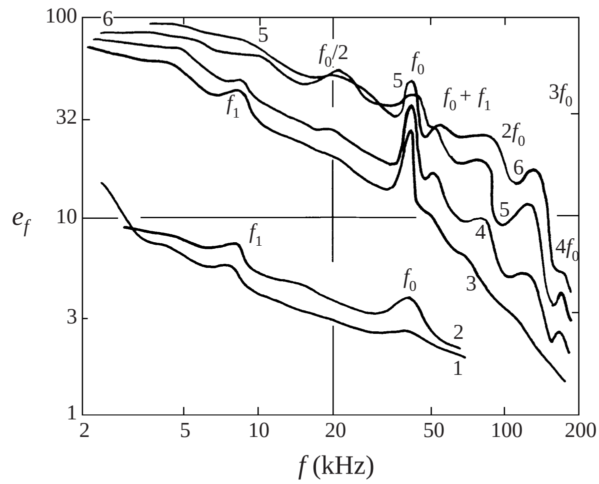

FIGURE 13. The energy spectra at $M_\infty = 2$, $l_k = 0$ and $y = y_{\langle e \rangle max}$ (curve 1, $x = 12$ mm; 2, 14; 3, 22; 4, 24; 5, 27; 6, 30).

decreasing $Re_{1\infty}$ are shown. This corresponds to results by McLaughlin (1971), in which using a hot-wire anemometer a significant fall in the level of disturbances was found in a wake with decreasing $Re_{1\infty}$.

The development of disturbances in the wake at $M_\infty = 2$ and $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$ is illustrated in figure 12, where the non-dimensional profiles of fluctuations $\langle \bar{e} \rangle = \langle e \rangle / \langle e \rangle_\infty$ (the normal coordinate y is measured from the plane of symmetry of the wake) for $x = 20$, 27 (shortly before transition), 44 and 60 mm (lines 1–4) are shown. The measurements were taken at the rather large wire overheating of 0.7, when the hot-wire anemometer actually registered the fluctuations of the mass flow. It is seen that in the laminar wake disturbances grow with increasing x; in addition the wake thickness remains about the same. However after the wake transition an appreciable flattening of the profile of disturbances occurs at the expense of a reduction in the maximum fluctuations in the wake. In addition, the thickness of the most disturbed part of the wake and the normal coordinate of the layer with the maximum disturbances increase more intensively than before transition. The development of the wake disturbances at $M_\infty = 2$ shown in figure 12 is the same as the results at $M_\infty = 4$ (§ 3).

In figures 13 and 14 the spectra of the energy of fluctuations in the hot-wire anemometer (the distribution of the amplitude of disturbances e_f for different frequencies f) in the layer with maximum (across the wake) values of $\langle e \rangle$, in the layer close to the critical one, respectively at $M_\infty = 2$, $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$ and $M_\infty = 4$, $Re_{1\infty} = 9 \times 10^6\,\mathrm{m}^{-1}$ are shown for different longitudinal coordinates. In figure 13 spectra at $x = 12$ and 14 mm (both in the free viscous layer) are plotted, together with those at $x = 22$, 24, 27 and 30 mm (the last one was obtained in the transitional zone); and in figure 14 spectra at $x = 40$, 45 and 60 mm (the last one was obtained in the transition zone) are shown. Similar spectra of energy of fluctuations were also obtained at $y = 0$ (in the plane of symmetry of the wake).

For $M_\infty = 2$ and 4 the well-known laws were followed. In the final part of the free viscous layer (directly before its transition into a wake) and in the zone of linear development of disturbances in the wake, the distinctive maximum in the spectral distribution of fluctuations appears – disturbances with frequency $f_0 = 43$ kHz for $M_\infty = 2$ and 35 kHz for $M_\infty = 4$ begin to grow significantly. The Strouhal number,

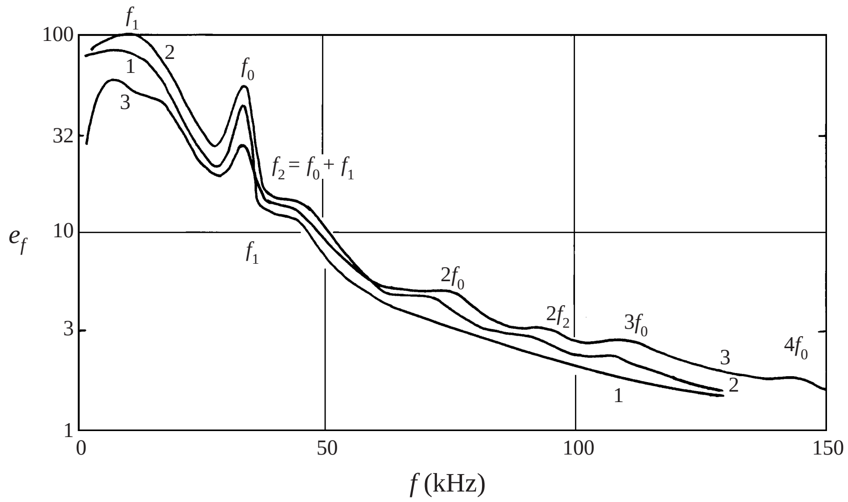

FIGURE 14. The energy spectra at $M_\infty = 4$, $Re_{1\infty} = 9 \times 10^6\,\mathrm{m}^{-1}$ and $y = y_{\langle e \rangle max}$ (curve 1, $x = 40\,\mathrm{mm}$; 2, 45; 3, 60).

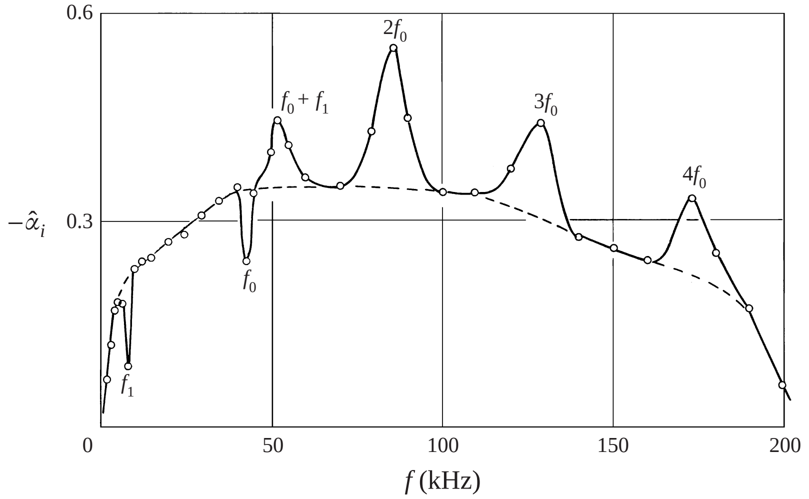

FIGURE 15. The wake disturbance amplification rates as a function of frequency at $M_\infty = 2$, $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$, $x = 23\,\mathrm{mm}$ and $y = y_{\langle e \rangle max}$.

based on the frequency of this maximum, the thickness of the wake near the throat and the speed of the undisturbed flow, $S = f_0 b_o / u_\infty = 0.3$.

In the nonlinear stage of development of disturbances in the wake (at about $22 < x < 29\,\mathrm{mm}$ for $M_\infty = 2$ and at $40 < x < 56\,\mathrm{mm}$ for $M_\infty = 4$) the basic tone with frequency f_0 still dominates in the spectrum of energy of fluctuations (down to the start of transition, when disturbances with frequency f_0 decrease, and the spectrum becomes more flat); however the nonlinear interaction of different fluctuations begins. In particular, disturbances with frequency $f_0 + f_1$ (f_1 being the frequency of the second maximum in the spectrum of energy of fluctuations in the linear stage) begin to grow significantly. Most likely, these two maxima are in resonance, and a triplet of waves (the third wave with frequency $f_0 + f_1$) appears. Also, the appreciable growth of harmonics with frequencies $2f_0, 3f_0, 4f_0$ begins. All this occurs against a background of appreciable moderation of the growth of disturbances with f_0 and f_1, which suggests that there is transfer of energy from the basic tone with f_0 and the disturbance with f_1 (or only from the basic tone) to the disturbance with frequency $f_0 + f_1$, and from the basic tone to the harmonics with frequencies $2f_0, 3f_0, 4f_0$. Figure 15, showing the dependence of the disturbance amplification rates on frequency for $M_\infty = 2, y = y_{\langle e \rangle_{max}}$, and the analogous dependence for $M_\infty = 4$ confirm this assumption.

The experiments described in this Section (and also the research on the determination of the transition position at $M_\infty = 4$, 5 and 7 in § 2) have shown the stabilizing influence of the growth of the Mach number: the disturbance amplification rates decrease, and the transition in the wake moves away from the model. The wake is also stabilized with decreasing unit Reynolds number. In the nonlinear stage of development of disturbances in the wake, harmonics were seen to grow.

5. Influence of the thickness of the flat plate and the length of its stern on the stability of a supersonic wake

The experiments were carried out in the wind tunnel T-325 at Mach number of free flow $M_\infty = 2$, unit Reynolds number $Re_{1\infty} = 5.7 \times 10^6\ \mathrm{m}^{-1}$ and the flow stagnation temperature of about 290 K. For measurement of the characteristics of stability and transition the same equipment as in § 4 was used. The test models were insulated steel symmetric flat plates of length (from the leading edge to the beginning of the stern part) $l = 61$ mm, thickness of 10 and 3 mm and width of 200 mm, the nose being a wedge with bevel half-angle of 14° and 0.1 mm bluntness. The changeable sterns of the plates had the shape of an opposite wedge: for the thin model, of thickness of 3 mm, the bevel half-angle was 14° (i.e. the same as the nose), for the plate of thickness 10 mm the bevel half-angles were 7°, 14°, and 90° (i.e. in the last case the back surface of the model was flat, the plate was bevelled to a right angle). The length of the stern part l_k (see the schematic in figure 16) had three different values: 40, 20 and 0 mm, and the ratio of the stern length to the plate thickness was $l_k/\Delta = 4$, 2 and 0 accordingly. Each plate was fixed rigidly to the lateral walls of the test section of the wind tunnel and were placed at zero angle of attack.

In figure 16 the dependence of the root-mean-square fluctuations of voltage on the longitudinal coordinate, obtained in the plane of symmetry of the wake, for the two plates of thickness 3 and 10 mm at $l_k/\Delta = 2$ are shown. The voltage fluctuations were non-dimensionalized by their maximum value, and the longitudinal coordinate x was measured from the beginning of the stern part. Curve 1 corresponds to $\Delta = 3$ mm ($l_k = 6$ mm), and curve 2 to $\Delta = 10$ mm ($l_k = 20$ mm). The vertical lines 3 and 4 mark the positions of the back edge of each model.

Figure 16 shows that with the increase in the thickness of the plate the disturbance amplification rates in the wake increase, which results in earlier transition. The figure shows that this change in position of the transition is unsignificant in ofterms the longitudinal coordinate x, but in terms the longitudinal coordinate $\Delta x = x - l_k$ (measured from the model back edge) the destabilizing influence of the increase in the plate thickness becomes more clear.

In figure 17 the non-dimensional profiles of the voltage fluctuations $\langle \bar{e} \rangle = \langle e \rangle / \langle e \rangle_\infty$ (the normal coordinate y was measured from the symmetry plane of the wake) for both plates at $x = 24$ mm are shown (curve 1 is for $\Delta = 10$ mm and $\Delta x = 4$ mm, and curve 2 for $\Delta = 3$ mm and $\Delta x = 18$ mm). They correspond to the initial zone of development of disturbances, when the fluctuations in the wake behind the thicker

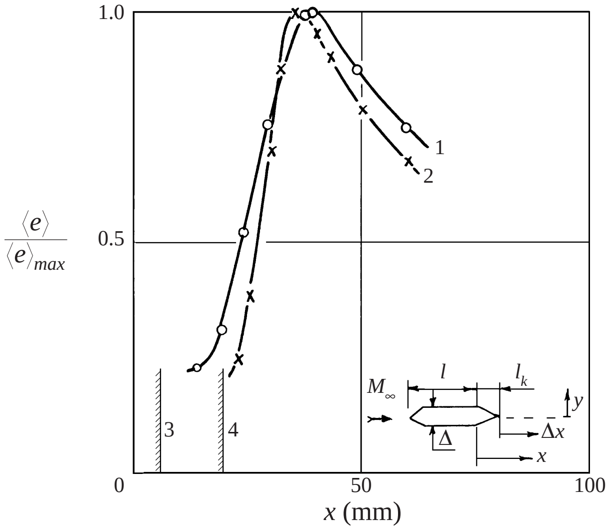

FIGURE 16. Distribution of the fluctuations along the longitudinal coordinate at $M_\infty = 2$, $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$ and $l_k/\Delta = 2$ (curves 1, 3, $\Delta = 3\,$mm, $l_k = 6\,$mm; 2, 4, $\Delta = 10\,$mm, $l_k = 20\,$mm).

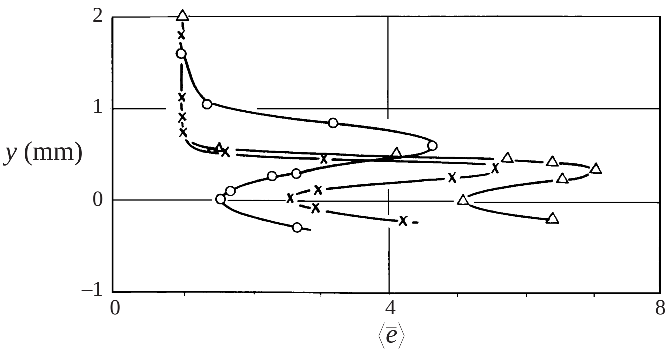

FIGURE 17. Profiles of fluctuations at $M_\infty = 2$, $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$ and $l_k/\Delta = 2$ (○, $\Delta = 10\,$mm, $x = 24\,$mm, $\Delta x = 4\,$mm; ×, $\Delta = 3\,$mm, $x = 24\,$mm, $\Delta x = 18\,$mm; △, $\Delta = 3\,$mm, $x = 36\,$mm).

plate have not yet 'overtaken' (as regards intensity) the similar disturbances behind the thin model. In the figure the profile of oscillations for $\Delta = 3\,$mm and $x = 36\,$mm (curve 3) is also presented.

In figure 18 the spectra of energy of fluctuations in the hot-wire-anemometer probe (the distributions of amplitude of disturbances e_f for frequency f) in both wakes are shown at $\Delta x = 7\,$mm (curve 1 corresponds to $\Delta = 3\,$mm and $x = 13\,$mm, and curve 2 to $\Delta = 10\,$mm and $x = 27\,$mm), i.e. at equal distances from the back edges of models. The measurements were performed in the layer with maximum (across the wake) values of $\langle e \rangle$. One can see that the disturbances behind the thicker plate develop more intensively than the oscillations behind the thin one. But figure 18 also shows that in both spectral distributions of fluctuations in the wakes there is a distinctive peak (at frequencies of 48 and 133 kHz respectively for the plates of thickness of 10 and 3 mm). The Strouhal number, based on the frequency of this maximum f_0, the wake

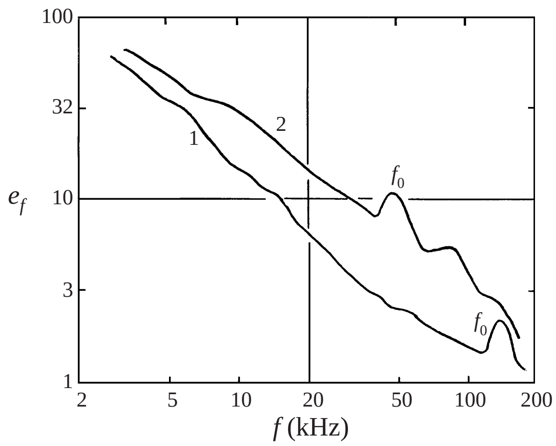

FIGURE 18. The energy spectra at $M_\infty = 2$, $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$, $l_k/\Delta = 2$ and $y = y_{\langle e\rangle max}$ (curve 1, $\Delta = 3\,\mathrm{mm}$, $x = 13$ mm, $\Delta x = 7\,\mathrm{mm}$; 2, $\Delta = 10\,\mathrm{mm}$, $x = 27\,\mathrm{mm}$, $\Delta x = 7\,\mathrm{mm}$).

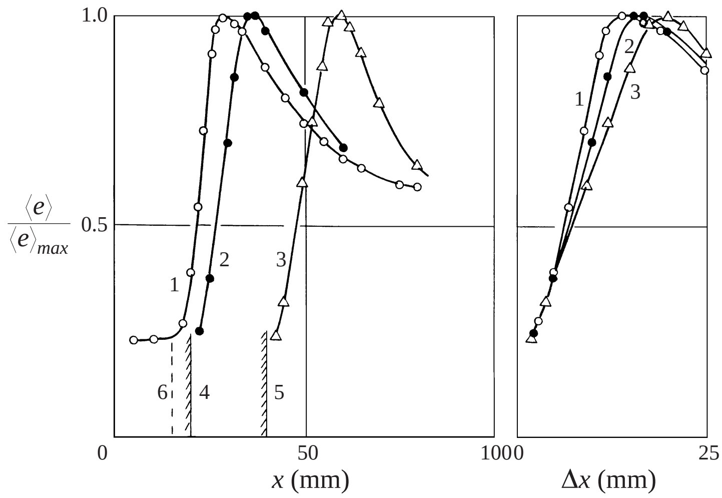

FIGURE 19. Distribution of the fluctuations along the longitudinal coordinates x and Δx at $M_\infty = 2$ and $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$ (curve 1, $l_k = 0$; 2, 4, $l_k/\Delta = 2$; 3, 5, $l_k/\Delta = 4$).

thickness near the wake throat (determined from the speed profile) and the speed of undisturbed flow, is equal to 0.3.

From figures 16 and 18 conclusions drawn about the increase of disturbance amplification rates in the wake with the increase in the thickness of a plate correspond to the results by Behrens (1968) in the wakes of cylinders at $M_\infty = 6$. And in the work by Behrens *et al.* (1971) on the wakes of wedges at $M_\infty = 4.5$ the model-thickness increase also resulted in earlier transition, as in the present experiments.

Then, a series of experiments on the influence of the length of the stern part (when it is an opposite wedge) of a plate on the stability and transition in the wake was carried out. The model of 10 mm thickness with changeable stern part was used. The length of the stern l_k had three values – 40, 20 and 0 mm ($l_k/\Delta = 4$, 2 and 0 respectively).

In figure 19 the dependence of root-mean-square voltage fluctuations in the plane of

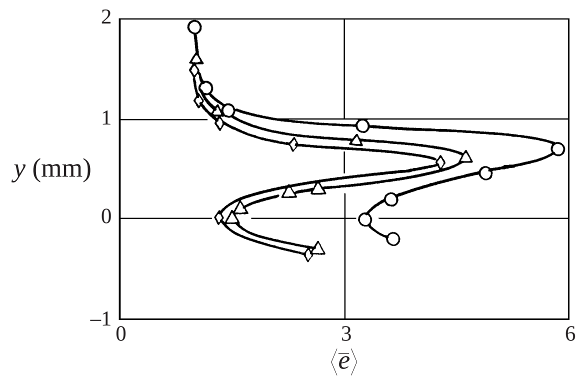

FIGURE 20. Profiles of fluctuations at $M_\infty = 2$ and $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$ (○, $l_k = 0$, $x = 24$ mm; △, $l_k/\Delta = 2$, $x = 24$ mm, $\Delta x = 4$ mm; ◇, $l_k/\Delta = 4$, $x = 44$ mm, $\Delta x = 4$ mm).

symmetry of the wake, non-dimensioned by their maximum values, on the longitudinal coordinate x are shown. Curve 1 corresponds to $l_k = 0$ (in this case there is a large recirculating zone behind the model), curve 2 to $l_k/\Delta = 2$, and curve 3 to $l_k/\Delta = 4$. The vertical lines 4 and 5 mark the positions of the back edges of the models with $l_k = 20$ and 40 mm respectively.

The curve $\langle e\rangle/\langle e\rangle_{max}$ =f(x) for $l_k = 0$ (in the large recirculating zone of length about 15 mm) corresponds approximately to the curve for the model with the stern as an opposite wedge and $l_k \approx 15$ mm (the dotted vertical line 6 on the figure corresponds to $l_k = 15$ mm). Therefore for comparison curves 1–3 on the left part of figure are shown in different coordinates on the right: $\langle e\rangle/\langle e\rangle_{max}$ and Δx: $\Delta x = x - 15$ mm is acceptable for curve 1.

Figure 19 shows that the wake-transition position moves back with the increase of the length of the stern part. In general, it is connected with the change in position of the wake throat and accordingly the coordinate of the beginning of the intensive increase of disturbances. But a comparison of the dependence of the intensities of the growth of fluctuations on the coordinate Δx for all three cases shows that the wake stability still increases a little (the disturbance amplification rates decrease a little). As the length of the stern part increases, the 'gradient' of the flow in the wake decreases, and the flow above the stern and directly in the wake approximates slowly to the flow above a flat plate. The transition in the boundary layer on the long flat plate at the present conditions of free flow begins at a longitudinal coordinate, measured from the leading edge, of not less than 160 mm.

In figure 20 the non-dimensional profiles of fluctuations are shown for three cases: 1: $l_k = 0, x = 24$ mm; 2: $l_k/\Delta = 2, x = 24$ mm, $\Delta x = 4$ mm; 3: $l_k/\Delta = 4, x = 44$ mm, $\Delta x = 4$ mm. That is, it is possible to compare in pairs curves 1 and 2 ($x = 24$ mm) and 2 and 3 ($\Delta x = 4$ mm). The comparison of curves 2 and 3 shows some delay in development of disturbances in the wake with the increase of length of the stern part.

In figure 21 the spectra of energy of fluctuations in the plane of symmetry of the wake ($y = 0$, figure 21a) and in the layer close to the critical layer with maximum (across the wake) values of $\langle e\rangle$ (figure 21b), are given. In figure 21(a) curve 1 corresponds to $l_k = 0, x = 27$ mm, and curve 2 to $l_k/\Delta = 2, x = 27$ mm, $\Delta x = 7$ mm. In figure 21(b) curve 1 is for $l_k = 0, x = 24$ mm; 2 for $l_k/\Delta = 2, x = 24$ mm, $\Delta x = 4$ mm; 3 for $l_k/\Delta = 4, x = 44$ mm, $\Delta x = 4$ mm; 4 for $l_k = 0, x = 27$ mm; and 5 for $l_k/\Delta = 2, x = 27$ mm, $\Delta x = 7$ mm. A comparison of curves 2 and 3 ($\Delta x = 4$ mm)

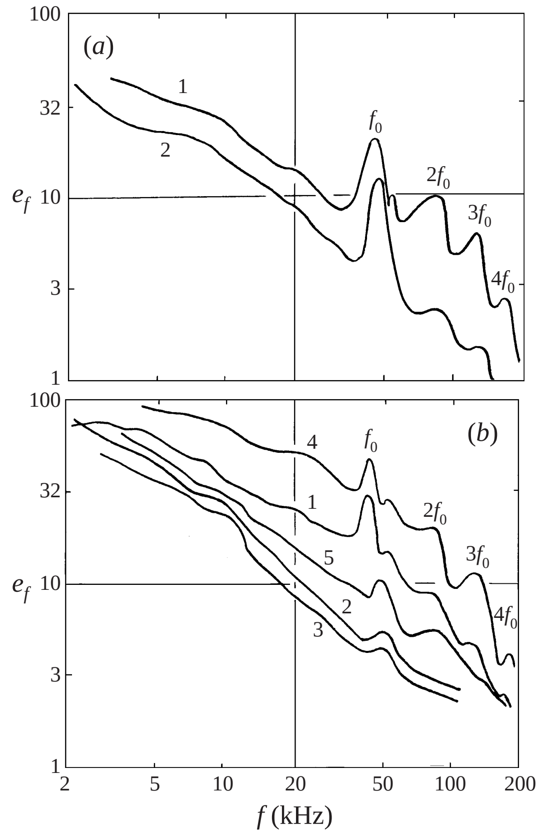

FIGURE 21. The energy spectra at $M_\infty = 2$ and $Re_{1\infty} = 5.7 \times 10^6\,\mathrm{m}^{-1}$: (*a*) for $y = 0$ (curve 1, $l_k = 0$, $x = 27\,\mathrm{mm}$; 2, $l_k/\Delta = 2$, $x = 27\,\mathrm{mm}$, $\Delta x = 7\,\mathrm{mm}$); (*b*) for $y = y_{\langle e \rangle max}$ (curve 1, $l_k = 0$, $x = 24\,\mathrm{mm}$; 2, $l_k/\Delta = 2$, $x = 24\,\mathrm{mm}$, $\Delta x = 4\,\mathrm{mm}$; 3, $l_k/\Delta = 4$, $x = 44\,\mathrm{mm}$, $\Delta x = 4\,\mathrm{mm}$; 4, $l_k = 0$, $x = 27\,\mathrm{mm}$; 5, $l_k/\Delta = 2$, $x = 27\,\mathrm{mm}$, $\Delta x = 7\,\mathrm{mm}$).

in figure 21(*b*) shows the increase of disturbances in a wake with decreasing stern length.

However, figure 21 is interesting for another reason also: it clearly shows the growth of disturbances that are multiples of the basic tone (with f_0) of frequencies $2f_0, 3f_0, 4f_0$ in the nonlinear stage of wake fluctuation development, i.e. the growth of harmonics is observed.

Thus, at Mach number $M_\infty = 2$ the development of disturbances in the wakes behind flat plates with symmetric wedge-shaped noses and sterns of different thickness and with different lengths has been investigated experimentally. It was found that the disturbance amplification rates in the wake increase with the growth of the model thickness, which results in earlier transition of a laminar wake to a turbulent one. It was also found that as the length of the stern part of the plate increases, the position of the wake transition moves back, and in addition the wake stability increases a bit (though very unsignificantly). A distinctive maximum (basic tone) was detected in the spectral distribution of fluctuations in the laminar part of the wake, and the value of the Strouhal number (based on the frequency of this maximum) of 0.3 is universal for plates of different thickness.

6. Development of artificial disturbances in the wake

At Mach number $M_\infty = 2$, together with A. D. Kosinov and Yu. G. Yermolaev, the development of artificial disturbances in the wake, caused by an electrical discharge on the surface of a plate, was investigated (Lysenko, Kosinov & Yermolaev 1998).

While there are many works in which the development of artificial disturbances in a supersonic boundary layer and a jet has been studied (in a boundary layer by Laufer & Vrebalovich 1960; Demetriades 1960; Kendall 1967; Kosinov & Maslov 1984; Kosinov, Maslov & Shevelkov 1986, 1989 1990*b,c*; Kosinov *et al.* 1990*a* and others; in a jet by, for example, Troutt & McLaughlin 1982; McLaughlin *et al.* 1975), only one similar study has been carried out in a shear layer (mixing layer) (Martens *et al.* 1994). In it only the spatial change of the front of a root-mean-square signal, caused by the discharge between a strip-electrode on the central body and central body itself, was studied. The basic achievement of that work is the result that for $M_1 = 3.9$ and $M_2 = 1.2$, at the two-dimensional position of the electrode, the front of the wave in a shear layer is turned downwards in the flow, i.e. it has been shown that there the growth of the oblique disturbances is greater than the amplification of the two-dimensional wave. However in that work the amplitude and phase spectra dependence on wavenumbers were not studied. Work on the development of artificial disturbances in a supersonic wake appear not to exist.

This part of the work contains a study, at supersonic free-flow velocities, of the development of artificial disturbances, initiated on the surface of a flat plate, in the system comprising the boundary layer on a flat plate, the boundary layer on an opposite wedge (at the stern) behind a fan of expansion waves, and the wake. The experiments were carried out in the wind tunnel T-325 at $M_\infty = 2.0$, unit Reynolds number $Re_{1\infty} = 5.4 \times 10^6\,\mathrm{m}^{-1}$, and flow stagnation temperature 290 K.

The model used was similar to that in § 5: of 80 mm length (from the leading edge), 10 mm thickness, 200 mm width, with bevel half-angle of the leading and back edges of 14°. The length of both the bow and stern parts was 20 mm. In the centre of the model a source of controllable disturbances (a discharge device similar to Kosinov *et al.* 1990*b*) was placed. The artificial disturbances penetrated into the boundary layer on the top surface of the plate 40 mm from both the leading and back edges, and it was from this point that the longitudinal $\tilde{x}$ and transversal z coordinates were measured.

The disturbances were excited by a high-frequency (20 kHz) electrical discharge of an alternating current between a screw-electrode with a sharp end-cone (being inside the model and isolated by ceramics from its metal parts) and the plate, and there was a round aperture of 0.4 mm diameter on the plate surface over the screw-electrode. On burning of the electrical discharge, fluctuations of pressure and temperature arose between the electrode and the model surface and disturbed the boundary layer, penetrating it through the aperture. At $\tilde{x} = 8$ mm, in the boundary layer, the excess of the maximum disturbance amplitude above the natural background was about 10, and in the wake it was about 2.

The electrical discharge circuit consisted of the generator of signals GZ-112/1, a capacity amplifier, a transformer, and electrodes (the circuit is described in Kosinov, Semionov & Shevelkov 1994). The maximum potential between the electrodes reached several kilovolts.

For measurement of disturbances the same equipment was used as in §§ 4 and 5. The selective amplifier U2-8 was used as a frequent filter. With its help the amplitude of a signal at frequency $f = 20$ kHz in a bandwidth of 1% was measured. The overheating of the probe wire was 0.8, and therefore it is possible to assert that the fluctuations of the mass flow were mainly accurately measured.

The measurements in the wake were carried out in the layer with constant average voltage in the diagonal of the 'bridge' of the hot-wire anemometer (it corresponded to measurements along a line of constant speed), equal to the average voltage in the layer close to the critical one in the boundary layer on the opposite wedge.

To determine the phase of the analysed signal in respect to the source of disturbances, a two-beam oscillograph S1-96, synchronizated with the discharge, was used. The process of burning of the discharge was controlled with the help of the oscillograph.

In the experiments oscillograms at several x sections were measured, and the amplitude of disturbances of the mass flow $A(z)$ and the difference in phases $\Phi(z)$ between the measured and synchronizing signals were determined for two values of $\tilde{x}$; the section $\tilde{x} = 0$ was the base.

The fluctuating and average characteristics of the flow were measured with the help of the automated measuring system of the wind tunnel T-325. The fluctuating and average hot-wire voltages were stored on a computer (DVK-3.2) using a ten-bit amplitude-digital converter (ADC) with 1 MHz reading frequency. The ADC was started synchronously with the generator setting the frequency of the introduced disturbances. Because of the increase of the signal/noise ratio 200 synchronous realizations of the signal were summed. The time length of a realization was 200 ms. The amplitude and phase of the disturbances at frequency 20 kHz were determined by discrete Fourier-transformation of averaged oscillograms. The averaged oscillograms of a fluctuation signal were controlled during an experiment, which allowed the bounds of an introduced wave packet in z to be determined rather precisely. The complete spectral processing of digital oscillograms was carried out by an IBM-486DX. For spectral processing of experimental data discrete Fourier-transformation was used (see Kosinov *et al.* 1994). From this the amplitude A_β and phase Φ_β of disturbances were determined (β is the wavenumber in the z-direction).

The phase speed of propagation of disturbances was defined as $C_x = \lambda_x f / U_e$, where $\lambda_x = 2\pi/\alpha_r$ is the wavelength of a disturbance (the wavenumber in the x-direction was defined from the relation $\alpha_r(\beta) = \Delta\Phi_\beta(x)/\Delta x$ because of the linear dependence $\Phi_\beta(x)$), and U_e was the flow speed on the edge of a layer.

Thus, the amplitude–phase field of the introduced disturbances at frequency $f =$ 20 kHz was determined, and the development of an initial monochromatic packet of Tollmien–Schlichting waves was studied.

In figure 22 the distribution of the amplitude of fluctuations A in the transversal direction z for $\tilde{x} = 18$ (the boundary layer on a flat plate), 30.1 (the boundary layer on an opposite wedge) and 48 mm (the wake) is shown (recall that the back edge of the plate and the beginning of the wake were placed at $\tilde{x} = 40$ mm). One can see that a monochromatic packet of Tollmien–Schlichting waves, which is rather narrow in the boundary layers on the flat plate and the opposite wedge, extends in the wake.

In figure 23 the amplitude A_β and phase Φ_β spectra at wavenumber β in the wake for $\tilde{x} = 43$ and 48 mm are presented (the distance from the model back edge was 3 and 8 mm respectively). In figure 23 (a) one can see that in the wake a discrete set of modes on β occurs. And figure 23(b) shows a great difference in increment of the phase Φ_β on x for different β. Since in the laminar wake the measurements were carried out at two x-sections only, the true difference between values of $\Phi_\beta(\beta)$ for $\tilde{x} = 48$ and 43 mm can increase (or decrease) by 360° compared to the values presented in figure 23(b). In any case, figure 23(b) shows that the phase speed $C_{x_{(\beta=0)}} > C_{x_{(\beta\neq 0)}}$. To study the wake stability another (in a system of coordinates driven with speed U_e) meaning of the phase speed $-C_R = (1 - C_x)/(1 - U_o/U_e)$ is used, where U_o is the speed of flow in

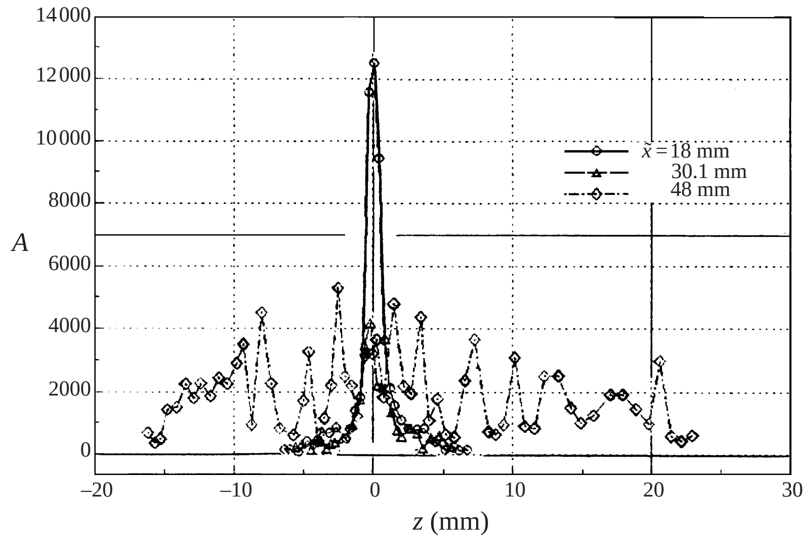

FIGURE 22. Distribution of the disturbance amplitude along the transversal coordinate at $M_\infty = 2$ for $\tilde{x} = 18$ mm (the boundary layer on the flat plate), 30.1 mm (the boundary layer on the opposite wedge) and 48 mm (the wake).

the plane of symmetry of a wake. From figure 23(b) it follows that $-C_{R_{(\beta=0)}} < -C_{R_{(\beta\neq 0)}}$. And since at $-C_R < 1/M$ (here $M = [U_e - U_o]/a$) the disturbances developing in the wake are subsonic, and at $-C_R > 1/M$ are supersonic (acoustic), from figure 23(b), which shows that for large β the increment of phase on x is great, it follows that in the wake acoustic disturbances are likely also to be present.

These results from figure 23 show that the disturbances entering the wake have complex wave structure. In Kosinov *et al.* (1989) it was found that acoustic disturbances and also regular disturbances with phase speed in the range $1 < C < 1 + 1/M$, called 'regular' by Mack (1965) because of the lack of singular points in the equations of stability, are added to the Tollmien–Schlichting waves on the cylindrical part of a 'cone-cylinder' model . It is quite probable that in the resent experiments the same mixture of different waves enters the wake. It should be noted that the section $\tilde{x} = 48$ mm was already in the zone of nonlinear development of disturbances in the wake (and at $\tilde{x} = 53$ mm, as oscillograms have shown, the wake was already transitional), and this also could be reflected in the shape of the spectrum A_β on β for $\tilde{x} = 48$ mm.

According to Lees & Gold (1964), in a two-dimensional wake both symmetric (varicose) disturbances, similar to symmetrically located (relatively a plane of symmetry) vortices, and antisymmetric (sinuous) disturbances, analogous to the Kármán vortex street, can develop. And according to Chen *et al.* (1990), in a flat supersonic wake for Mach number 2 the two-dimensional waves (with angle of the inclination $\chi = 0$) amplify most from the antisymmetric disturbances, and the oblique waves with $\chi = 50°$ most from the symmetric disturbances. From figure 23 it is seen that in the present experiments in the wake the three-dimensional disturbances with $\chi \sim 30°–70°$ amplified more than others, which testifies indirectly that in the experiments discussed here the symmetric (varicose) disturbances developed most in the wake.

Thus, it is possible to assume the following picture of the development of disturbances in the system comprising the boundary layer on a model and a wake. The

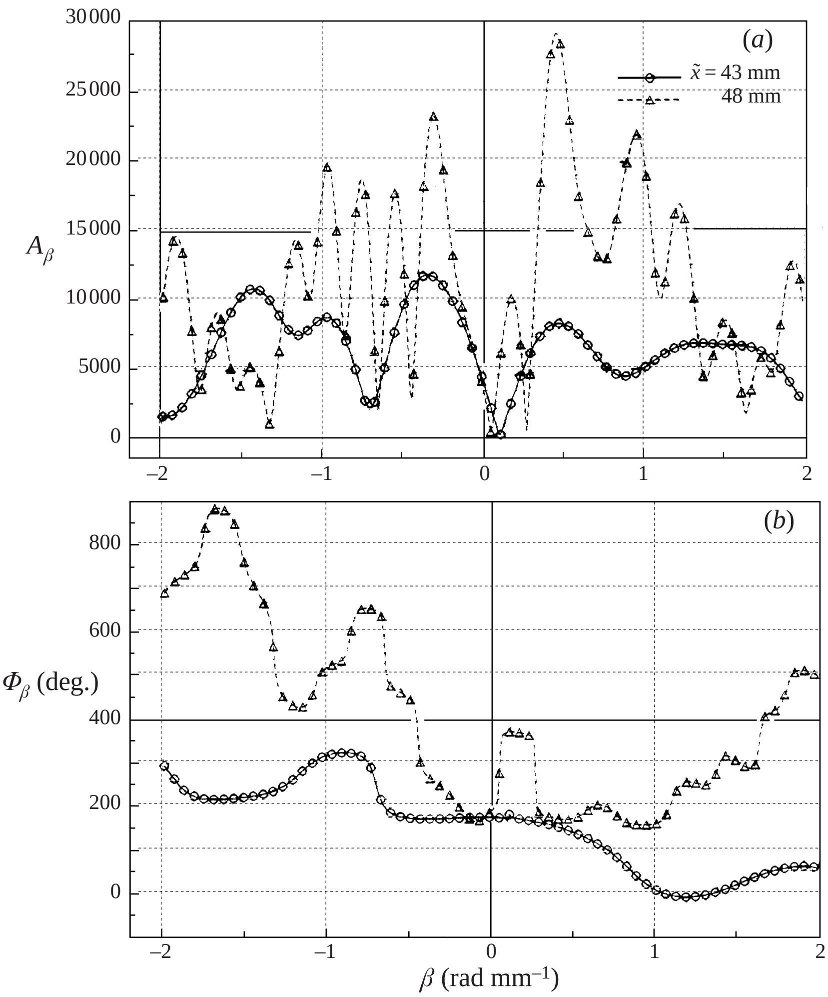

FIGURE 23. (a) The amplitude and (b) the phase spectra (for wavenumber β) in the wake at $M_\infty = 2$ for $\tilde{x} = 43$ ($x = 23$) mm and $\tilde{x} = 48$ mm.

disturbances developing on the stern with an opposite wedge have very complex wave structure – a mixture waves of different types. And the whole of this mixture enters the wake. The disturbances in the wake develop according to the theory of the stability of free shear flow, which originally (without the consideration of viscosity) is unstable. To meet the conditions of the present experiment, in general symmetric disturbances develop in the wake, and thus the problem of the receptivity of a wake becomes very interesting. It is very important to know which disturbances from the mixture entering a wake will be transformed by the wake into eigen ones, and what their coefficient of amplification will be.

7. Basic conclusions

The following conclusions can be drawn.

The condition (laminar, transitional or turbulent) of the boundary layer at the end of a model exercises an influence (by means of the change of speed profiles and other parameters, and also at the expense of a change of the level of disturbances) on the position of the transition in the wake, though the transition can occur simultaneously both in the boundary layer and in the wake (because of corresponding instabilities).

Compressibillity of the flow (increasing Mach number) stabilizes the wake disturbances – their amplification rates decrease, and the transition moves away from the model.

Cooling of the model surface at $M_\infty \sim 7$ exercises a destabilizing influence on the development of disturbances in the wake.

With increase of unit Reynolds number the beginning of transition in a wake moves forward to a rear critical point.

At Mach numbers $M_\infty = 2$ and 4 the development of disturbances in the free viscous layer and in the wake behind the flat plate with a symmetric wedge-shaped nose (with a sharp leading edge) and blunt (bevelled to a right angle) stern was investigated. The development of wake disturbances was traced both in its linear and nonlinear stages of development. The characteristics of stability of the flow in the free viscous layer and wake have been obtained.

It was confirmed that a distinctive maximum appears in the spectral distribution of fluctuations, corresponding to a Strouhal number (based on the frequency of this maximum) of 0.3.

With an increase in the model thickness the disturbance amplification rates in the wake increase, which results in earlier transition of a laminar wake into a turbulent one.

With an increase in the length of the plate stern, the position of the wake transition moves back, while the wake stability rises a little (though very unsignificantly).

In the nonlinear stage of development of disturbances, the occurrence of a triad of waves, satisfying a resonant correlation of frequencies, and the growth of harmonics are observed.

It was for the first time possible to introduce artificial disturbances into a supersonic wake.

A monochromatic packet of Tollmien–Schlichting waves, rather narrow (in the transversal coordinate) in the boundary layers on the flat plate and an opposite wedge, was found to extend in the wake. The wake disturbances have a complex wave structure.

At Mach number of free flow 2.0, the three-dimensional disturbances are the most unstable in the wake.

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